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An Empirical Application

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ABSTRACT

Several studies claim that people have a tendency to be overoptimistic (Coelho; 2010; Lovallo & Kahneman, 2003). Furthermore, some researchers suggest that optimism could be prevalent in managers as a result of the selection process (Heaton, 2002). Nevertheless, there is very little literature about the subject of optimism and managerial decisions (Coelho, 2010). In this study we present a frontier model of expectations with an optimistic bias based on the adaptive expectation model. In our framework, optimism is considered as a positive random term which skews expectations from a normal forecast based on rational assumptions. We model investment decision based on expectations about key variables such as sales or cash flow. We posit that managers have a skewed viewpoint of reality.

An application of the empirical model in the context of the American retail industry is provided. This paper contributes to increasing the literature about unrealistic optimism as well as applying productivity and efficiency techniques in the management field.
1 Introduction

Expectations are the cornerstone of the decision-making process. It is safe to claim that people usually make decisions based on their ideas about the future. Expectation formation has been the subject of analysis of a diverse array of disciplines. There are several theories that try to explain how individuals make forecasts about future events. For example, expectations can be the result of an adaptive adjustment, where predictions are based on the most recent values of a variable. Expectations can be formed just as the economic theory predicts, using all the available information. There is a wide range of different concepts about how human beings make predictions.

This study is based on the adaptive expectations model. Expectations are generated based on the most recent mistakes. We modify the original adaptive expectation model to include the possibility of a positive systematic bias and we offer a new interpretation of the stochastic frontier model inefficiency term. In this context, the inefficiency term measures optimism. Our hypothesis is based on the growing literature about the prevalence of overoptimism among decision-makers. We modeled investment decisions based on predictions about future sales in the American retail industry.

We posit that managers make systematic errors when they create their expectations about the future. Specifically, managers overestimate future performance. In statistical terms, we claim that the prediction error term has a positive mean. Overoptimistic behavior could be potentially detrimental to the company’s performance. Several authors (Coelho, 2010; Hackbarth, 2008 and Heaton, 2002) have stated that the issue of optimistic bias has not been studied in depth. Coelho (2010) claims that “positive illusions create distortions which may be the most important source of efficiency loss in the economics systems, and as yet their policy implications may be ignored.” On the other hand, being overoptimistic can be considered rational (Van den Steen, 2004). The explanation offered by Van den Steen (2004) is similar to the winner’s curse. People tend to choose the actions that they consider more likely to happen. Although excessive optimism can be associated with underperformance, there is no direct connection. Choice-driven overoptimism does not rule out the possibility that best performers are excessively optimistic as they correct their estimates through time.1

Our hypothesis differs from the rational expectations framework. We do not consider that on average the difference between the observation and the anticipated value is zero  

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1 In addition, overconfident managers could increase their level of confidence as they obtain more data. See Van den Steen (2011).
Historically, the rational expectation theory has been tested using survey information (e.g. Lovell, 1986; Levine 1993 and Benitez-Silva & Dwyer 2003). These surveys seek to “observe” people’s expectations. The analysis of the survey contrasts these expectations with the actual realizations of the anticipated variables in order to verify rational expectations hypothesis. We do not have information about these expectations. Instead, our methodology is based on the assumption that managers make positive systematic biases in their predictions and tests whether or not this assumption is correct.

We use a dataset with the main discount retail chains (Walmart, Target, Kmart, Sears and May). We have two objectives: first, we want to verify that optimistic bias exists, by calculating an LR rest on whether the biased error term is equal to zero or not; the second objective is to observe what kind of companies exhibit the largest systematic biases: the successful firms (Walmart and Target) or the companies that failed or had poor performance (Kmart, Sears and May). Our methodology requires a grid search using the Maximum Likelihood Estimation (MLE). To our knowledge, this has been done using OLS (e.g. Hansen, 1999; Yélou et al. 2010) but not with MLE. This implies an additional level of difficulty.

The rest of the paper is organized as follows: section 2 provides a brief literature review about the topic of excess optimism; our model is presented in section 3; the dataset is described in section 4; results are analyzed in section 5, and section 6 contains the conclusions.

2 Literature Review

Excess optimism or unrealistic optimism was first studied in the psychology field. In the Journal of Applied Psychology, Larwood and Whittaker (1977) published the results of several experiments aimed at demonstrating that optimistic bias exists and that it leads to overestimating organizational performance, in particular sales volume. They state that this bias is reduced if the agents have failed in their earlier forecasting experiences but it remains high despite being advised to be “realistic.” Weinstein (1980) carried out a very important study on the subject of unrealistic optimism in the social science field. The author defines unrealistic optimism as the tendency to assign low probability to negative events and high probability to positive events. Weinstein (1980) lists two possible sources of unrealistic optimism. The motivational explanation describes excess optimism as the byproduct of defensiveness or wishful thinking. On the other hand, this irrational bias could be the result of a cognitive flaw. For example, people can overlook the similarities with respect to others and
assume that the likelihood of an extreme event is different from the general population (extreme probability bias). Furthermore, agents could be unfamiliar with the assessed event or have the illusion of control. Coelho (2010) claims that motivational circumstances or cognitive bias seem to be more prevalent in the managerial population.

Roll (1986) was one of the first to study unrealistic optimism with respect to investment behavior. The author analyzed why mergers and tender offers fail to deliver the expected results. Roll (1986) claims that a manager’s evaluation of future acquisitions could be the result of manager’s hubris, which is a presumption that his/her assessment is more accurate than the market valuation. An interesting aspect of Roll’s framework is that he considered managers’ valuation as a random variable the left tail of which is never observable. Managers’ assessment would only be observable if the assessment is higher than the average, which is the market valuation. His insights are similar to the approach taken in this study. We model excess optimism as a positive half-tail random error.

There is increasing evidence that capital structure decisions are very sensitive to the presence of overoptimistic bias. The idea is that “irrational managers” perceive external funds as excessively expensive and prefer to use internal funds instead. Irrationality is defined as having unrealistic optimism or being overconfident. Overconfidence is excessive confidence in the precision of a forecast and it is related with optimism. It has been stated that irrational managers prefer free cash flow than debt or equity (Heaton, 2002; Malmendier and Tate, 2005) and prefer debt than equity (Hackbarth, 2008) if they hold an optimistic bias. Managers’ distorted perception makes them overestimate the returns of their projects. Therefore, if they have access to internal funds they probably could undertake projects with a negative net present value. On the other hand, if managers lack internal funds, they may reject projects with positive net present values because they consider external funds costly.

The relationship between optimism and firm value has been characterized as non-monotonic (Hackbarth, 2008). A similar finding was obtained when overconfidence levels were analyzed (Goel and Thakor, 2008). In general, shareholders would prefer optimistic rather than rational managers. Nevertheless, for extreme values of optimism the relationship is found to be negative. The reasoning behind these findings comes from the risk averse nature

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Coelho (2010) states that researchers adopt different definitions for the terms overoptimism and overconfidence in literature. In this study we express overoptimism as the positive bias in the prediction of a future variable. We consider our definition to be equivalent to that of Weistein (1980).

Hackbarth (2008) distinguishes between optimism and overconfidence. He found that optimistic managers prefer debt than equity but overconfident managers prefer the opposite. Overconfident managers underestimate the risk levels of a project and consider that equity is overvalued.
of managers. Risk averse managers underinvest meanwhile managers with overconfidence or optimism select higher levels of investment which are closer to the optimal values for the shareholders. After a certain threshold the overinvestment is detrimental to the company’s value. Furthermore, moderately optimistic managers could reduce principal-agent conflicts because the high debt levels constrain them to use discretionary funds (Hackbarth, 2008).

Goel and Thakor (2008) argue that the internal selection process of a company favors irrational, and in particular overconfident, managers. Internal tournaments might encourage managers to take more risks (Heaton, 2002). Since overconfidence makes the agent underestimate risks, people with this trait are more likely to be chosen than those who are rational. Therefore, “overconfidence is likely to be a more prevalent attribute than in the general population.” (Goel and Thakor, 2008; p. 2739).

Besides investment decisions, unrealistic optimism and overconfidence has been studied regarding entry decisions, (Camerer and Lovallo, 2003) and search behavior (Papenhaussen, 2010). It has been found that distorted perceptions of self-skills encourage an excess of entry in competition. The effect is even larger when agents know a priori that their chances of success depend on their skill levels (reference group neglect). These findings could explain why people choose performance-based incentives more than expected. Regarding search behavior, moderately optimistic managers put more effort into searching for a solution than rational agents. However, if there is a considerable excess optimism, managers might choose to do nothing and wait for the solution to arrive. Once more, the effect of optimism seems to be non-monotonic.

Rational expectations:

According to Muth (1961), the average expectations in an industry are as accurate as elaborated equation systems. This author is the precursor of the rational expectation theory. He asserts that firms’ expectations of the future are distributed similarly to what the economic theory would predict. Although firms make mistakes in their forecasts, the mean error is equal to zero. Moreover, it is also assumed that it is not a waste of information. These assumptions exclude the possibility of a systematic bias by the decision-maker since this would imply that he/she has not used all the available information to correct his/her expectations.

Some tests have been developed to validate the rational expectation theory predictions (Maddala, 2001). These tests are based on information collected through surveys. Lovell (1986) analyzes some of the empirical evidence about rational optimism. He illustrates that in
some studies on forecasting inventory needs based on sales, some companies are chronically overoptimistic while others are pessimistic. However, the overestimation of the overoptimistic firms cancels out the underestimation of the pessimistic firm; thus the general picture represents a scenario with no bias. Nevertheless, at the individual level, the rational expectation theory was not corroborated. The author explains that there are two versions of rationality. Weak rationality requires the error measurement to not be correlated with past values of the forecasted variable. On the other hand, the strong rationality assumption imposes no correlation of the error term with all the information available for the decision-maker. Lovell (1986) reports, that in Hirsh and Lovell (1969), weak rationality is not satisfied. Furthermore, the author reviews other works on rationality tests in subjects such as inflation, wages, national accounts, budget, and EPA mileage. In most of these studies, the rationality hypothesis is rejected or the evidence is inconclusive.

More recently, Benitez-Silva and Dwyer (2003) studied the rational expectation hypothesis using micro-data such as retirement age, health, employment and income, among others. The results of their research do not reject the rational expectations hypothesis after controlling for measurement errors and sample selection biases. In the management field, Levine (1993) analyzes whether corporate executives hold rational expectations using survey data. The difference from previous studies is that managers paid money for participating in the study and were interested in the results. Levine (1993) argues that this characteristic answers the criticisms about testing rationality. It has been stated that participants in these surveys are not truthful and accurate in their responses. The results reject the rational expectation hypothesis. We found it interesting that managers seemed particularly optimistic. For instance, it was reported that if managers predicted 8% market growth, the market would actually grow by 2%. Another example was the price forecast; if the managers predicted a 5% increment in their output prices, in reality prices would have increased by 0.5%. Furthermore, Levine (1993) shows that managers put too much importance on the most recent observation instead of taking into account the entire history. Nonetheless, the author tests other model specifications including the adaptive expectation models. All of these specifications are rejected as well.

Finally, Van den Steen (2004) proposes that overoptimism could be considered a rational choice. Instead of relying on an unobserved mechanism to explain this behavior (such as a cognitive flaw or motivational theories), Van den Steen provides a theoretical model where agents’ optimal choices make them overoptimistic. These agents choose those actions
that have a higher probability of success. Nevertheless, they have different prior assumptions, thus the agent will choose those actions with an overestimated subjective probability. It is not explicit whether overoptimism will imply poorer performance or not.

*Expectation Formation*

The starting point of our empirical background is the adaptive expectation model. According to Begg (1982) the adaptive expectation model was introduced by Cagan (1956) and Nerlove (1958); although, Evans and Honkapohja (2001) and Maddala (2001) claim that the origin can be traced back to Fisher (1930). In simple terms, the adaptive expectation theory states that people revise their expectations based on previous forecasting mistakes (Attfield, Demery and Duck, 1991).

Attfield et al. (1991) explain that there are three advantages to the adaptive expectation model. First, the theory implies that people could have wrong expectations in the short run but not in the long run. The second “attractive feature” is that this theory can be used in different contexts such as GDP growth, unemployment rate and interest rate, among others. In this study we focus on sales forecasts. The third feature is that it relates the current expectations of a variable to the past values of this variable.

One important issue that we need to clarify is who the predictor is. The adaptive expectation model implies that expectations are formed based on the past values of the analyzed variable. Hence, if we claim that the predictor forms their sales expectations based on past values of this variable we are implicitly stating that these predictors “remember” sales values from a long time ago when they make their forecasts about the future. Nevertheless, as we will explain in the next section, the adaptive expectations model imposes geometric declining weights as the variable goes back in time. Therefore, the most recent observations are relevant in determining current expectations and very old information contributes insignificantly in the formation of these expectations as Attfield et al. (1991) pointed out.

The adaptive expectation models in macroeconomics assume that the coefficients of past information represent averages of all the agents involved in the economic process. Similarly, in our application, these coefficients correspond to the market assessment. Therefore if two firms have exactly the same past sales history, they would have the same forecast for future sales if there is no unrealistic optimism bias. Consequently, in our study, managers with overoptimistic bias deviate from the market prediction and this deviation is modeled by adding a positive bias error term to the market expectation.
In this study, we define excessive optimism as the error made by managers in the process of expectation formation. This error has a right half-tail distribution and an average close to zero. The reason justifying these conditions is the presumed characteristics of the managers identified in the previous literature. Optimism bias seems to be a prevalent attribute of managers; pessimism or rationality are not traits that shareholders promote in a managerial team. It is difficult to imagine a scenario where managers expect to perform below the industry average and remain in their positions for a long time. Even in the situation that exogenous variables such as economic or social conditions alter future expectations negatively; managers’ self-confidence in their skills would make them believe that they could handle the critical condition much better than rationality would imply. Furthermore, our definition of optimism is in keeping with the “unrealistic optimism” proposed by Weinstein (1980). Positive events such as a higher sales volume would be presumed to be more likely than a low sales volume.

In the next section we will further describe the empirical model applied in this study.

3 Empirical background

Consider the following equation:

\[ y_{i,t} = a + b x_{i,t+1}^* + \epsilon_{i,t} \]  \[1\]

Where \( y_{i,t} \) stands for firm \( i \) investment in period \( t \), \( x_{i,t+1}^* \) is the firm’s expected sales during period \( t+1 \) and \( \epsilon_{i,t} \) is a zero-mean symmetric error term.

We assume that these expectations are formed using, partially or entirely, past history. Hence, we adopt a traditional Adaptive Expectation Model to model expectations and assume that:

\[ x_{i,t+1}^* = \beta_0 x_{i,t} + \beta_1 x_{i,t-1} + \beta_2 x_{i,t-2} + \cdots + \beta_k x_{i,t-k} \]  \[2\]

This model is called *distributed lag model of expectations* since it uses a weighted average of past values of the forecasted variable to summarize the formation process of expectation implied in the data. Several naive models of expectations are nested in [2]. For instance, if we assume that \( \beta_0 = 1 \) and the remaining coefficients are zero, we get a model in

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\[ \text{This section is mostly inspired by Maddala's (2001) textbook.} \]

\[ \text{In this study we use capital as a proxy for investment. With the information we have on investment we get a correlation coefficient of 0.84. We did not use investment directly because of problems of convergence.} \]
which the expected sales will be equal to the current sales. On the other hand, if we assume that $\beta_0 = 2$, $\beta_1 = -1$ and the remaining coefficients are zero, we obtain a model in which it is expected that future sales will increase by the same quantity as the latest increase.

The model in [2] is called a finite distributed lag model since the number of lagged past values is finite. Koyck (1954) suggested using an infinitive lag distribution with geometrically declining weights. In this case, the deterministic relationship between expectation and past values can be written as:

$$x_{i,t+1} = \sum_{k=0}^{\infty} \beta_k x_{i,t-k}$$  \[3\]

Where $\beta_k = \beta_0 \lambda^k$ and $0 < \lambda < 1$. If the sum of the infinitive series is $\beta_0 / (1 - \lambda)$ and this sum is equal to one we get:

$$x_{i,t+1} = \sum_{k=0}^{\infty} (1 - \lambda) \lambda^k x_{i,t-k}$$  \[4\]

It is straightforward to get the following relationship:

$$x_{i,t+1} - \lambda x_{i,t} = (1 - \lambda) x_{i,t}$$  \[5\]

This equation can be written equivalently as:

$$x_{i,t+1} - x_{i,t} = (1 - \lambda) (x_{i,t} - x_{i,t})$$  \[6\]

This equation says that expectations are revised based exclusively on the most recent error. For this reason the model above is called an adaptive expectations model. Imagine that $\lambda = 0.5$, in this case future expectation, will be the sum of the previous expectation plus 50% of the previous forecast mistake. If we lag equation [1] by one period and multiply throughout by $\lambda$, we get

$$\lambda y_{i,t} = \lambda a + \lambda b x_{i,t+1} + \lambda e_{i,t}$$  \[7\]

Subtracting equation [7] from [1], and after some straightforward manipulations, the equation to be estimated can be written as:

$$y_{i,t} = \alpha + \beta x_{i,t} + \lambda y_{i,t-1} + (e_{i,t} - \lambda e_{i,t-1})$$  \[8\]

Where $\alpha = (1 - \lambda) a$ and $\beta = (1 - \lambda) b$ are parameters to be estimated. This model cannot be estimated directly by ordinary least squares (OLS) because $y_{i,t-1}$ is correlated with an error term that is autocorrelated as well. This problem could be avoided by using the
instrumental variables method as long as valid instruments for $y_{i,t-1}$ are found. An alternative strategy is using an OLS estimator combined with a grid search over the $\lambda$ parameter. In this case, the model is estimated in two stages. In the first stage, given a particular value of the $\lambda$ parameter, the remaining parameters are estimated by OLS. The next step requires the residual sum of squares $RSS$ under the estimated parameters. The value of the $RSS$ is also a function of $\lambda$ because the estimated parameters are functions of $\lambda$. Since $\lambda$ is unknown, it must be estimated from the data set. We might choose the value of $\lambda$ for which $RSS(\lambda)$ is the minimum, that is:

$$\hat{\lambda} = \arg\min_{0 < \lambda < 1} RSS(\lambda)$$

A model of expectations with excess optimism

In the previous section we have modeled managers’ expectations as a deterministic function of past values of firm sales. Two comments are in order regarding this relationship. First, as all parameters of the expectation function [4] are common to all firms in the market, two firms would receive the same prediction if they shared the same past information. Therefore, we can interpret this function as the “normal” expectation that a particular firm would receive in the market given its own past history. Second, as the adaptive expectation model is unbiased, we have implicitly assumed in the previous section that firm managers are efficient in the sense that they do not make systematic mistakes when forming their expectations. However, a scenario characterized by “excess optimism” might be possible, in the sense that managers’ expectations are persistently higher than normal. This situation can be incorporated into our model by modifying the equation [4] as follows:

$$x_{i,t+1}^* = z_{i,t+1}(\lambda) + u_{i,t}^+$$

Where $z_{i,t+1}(\lambda) = \sum_{k=0}^{\infty} (1 - \lambda)^k x_{i,t-k}$ denotes the deterministic relationship between expectation and past values, and $u_{i,t}^+ \geq 0$ is a non-negative random term capturing the excess optimism. We use $\lambda$ because $z_{i,t+1}$ depends on this parameter. Since $u_{i,t}^+$ is not observed it is assumed to be random following one of the one-sided distributions traditionally used in the stochastic frontier literature, e.g. half-normal distribution. A reason for $u_{i,t}^+$ to

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6 For instance, we can use in this framework $x_{s,1}$ as instrument for $y_{s,1}$.

7 A similar two-stage model that involves a search procedure has been used, for instance, in Hansen (2003) and Yéloa et al. (2010).

8 It is worth noting that in this literature an equation like (10) is equivalent to a deterministic frontier function because the function to be estimated ignores other determinants of expectations that are observed by
follow a one-sided distribution is that managers are required to make the company perform at least as well as the average performance of the industry. This requirement is even more pertinent for publicly traded companies. If a manager is perceived as unsure about their ability to perform better than the market, then the shareholders would replace the manager.

We also expect that $u_{i,t}^+$ is asymmetrically distributed where high levels of excess optimism are less likely because most managers in a particular market do not make decisions based on unsustainable expectations, and they are used to sticking to the normal expectations in the market.\(^9\) This asymmetry assumption plays a critical role in our model because we precisely take advantage of the asymmetry (skewness) of the excess-of-optimism term to identify firms with unsound expectations that might go bankrupt in the future.\(^10\)

In this context, testing that this non-negative random term exists is equivalent to testing the existence of excess optimism or upward-biased expectations. Hence, this test resembles the so-called "tests for rationality". These tests assume that both current data and predictions are available, and test whether predictions are unbiased ex post. This cannot be done in our application, as managers’ expectations are not observed by researchers. We use a different approach. Our test endeavors to examine whether expectations are (upward) biased by modeling ex ante the existence of these potential biases in the data generating process.

The model in [10] can be considered as a frontier model where the dependent variable (i.e. firm manager’s expectations) is not observed by researchers. What we do observe are the consequences of these expectations throughout the investment equation [1].

Regarding the alternative estimation strategies, it should be noted that equation [5] can be written in a scenario characterized by excess optimism such as:

$$x_{i,t+1}^* = \left[z_{i,t+1}(\lambda) + v_{i,t}\right] + u_{i,t}^+$$

where $v_{i,t}$ is a random term capturing other determinants of expectations that is conventionally assumed to be distributed as a normal random variable with zero mean. The term in brackets is equivalent to a stochastic frontier function because the function to be estimated is stochastic as it takes into account unobservable factors that determine managers’ expectations. It can be shown that the final equation to be estimated does not change if we use a stochastic expectation frontier function, except that the error term in this equation is actually the sum of two random terms, $\epsilon_{i,t}$ and $v_{i,t}$ that cannot be distinguished because both are symmetrically distributed. For this reason, we will assume hereafter that there are no other determinants of expectations, except the firm-specific past values of profits or sales.

\(^9\) Obviously, this is correct except in "bubble" situations where overall market expectations are also unsustainable.

\(^10\) The empirical strategy to distinguish the one-sided random term from other random terms in the model when the one-sided term is also symmetrically distributed is an issue that, nowadays, is at the center of a heated debate among researchers in the stochastic production frontier area of research (see, for instance, the proposals presented in the last EWEPA conference held in Pisa).
\[ x_{i,t+1}^* - \lambda x_{i,t}^* = (1 - \lambda)x_{i,t}^* + (u_{i,t}^* - \lambda u_{i,t-1}^*) \]  

And hence the equation (8) to be estimated takes the following form:

\[ y_{i,t} = \alpha + \beta x_{i,t} + \lambda y_{i,t-1} + \tau_{i,t} + b(u_{i,t}^* - \lambda u_{i,t-1}^*) \]  

Where \( \tau_{i,t} = \varepsilon_{i,t} - \lambda \varepsilon_{i,t-1} \) is a symmetric (but auto-correlated) random term with zero mean, the last term in [12] is the difference between two one-sided random terms, the distribution of which is not known. Wang and Ho (2010) face the same problem, though in a different context, and propose using a one-sided random term that satisfies the so-called scaling property.\(^{11}\) This property allows us to get a tractable likelihood function. Indeed, let us assume that the non-negative random term capturing the excess optimism can be written as:

\[ u_{i,t}^* = g(t, \theta) \cdot u_i^* \]  

Where \( g(t, \theta) \) is a deterministic function of time and \( u_i^* \) is a time-invariant one-sided random term.\(^{12}\) In this case, we can rewrite the last term in [12] as follows ignoring the parameter \( b \):

\[ u_{i,t}^* - \lambda u_{i,t-1}^* = [g(t, \theta) - \lambda g(t - 1, \theta)] \cdot u_i^* = H(t, \theta, \lambda) \cdot u_i^* \]  

And placing [14] in [12] we get the final equation to be estimated:

\[ y_{i,t} = \alpha + \beta x_{i,t} + \lambda y_{i,t-1} + \tau_{i,t} + bH(t, \theta, \lambda) \cdot u_i^* \]  

The distribution of \( u_i^* \) is not affected by the transformation, thus the whole model can be estimated by maximum likelihood. This model is similar to that introduced by Wang and Ho (2010) except for the first-differencing transformation of the variables. While these authors used pure first-differences of the variables, in our application we use a partial first-difference since for each variable we do not subtract the total value of the lagged variable. In this sense, while Wang and Ho (2010) need to assume that the scaling function \( g(\cdot) \) is not constant in order to make the likelihood tractable, our model can be estimated even when optimism is time-invariant.\(^{13}\)

\(^{11}\) A discussion of the advantages of this property can be found in Wang and Schmidt (2002) and Álvarez et al. (2006).

\(^{12}\) Particular functional forms for \( g(\cdot) \) have been proposed by Kumbhakar (1900), Battese and Coelli (1992), and Orea and Kumbhakar (2004).

\(^{13}\) Indeed, if we assume that \( (t, \theta) \equiv g(t - 1, \theta) = 1 \), then \( H(t, \theta, \lambda) = H(\lambda) = 1 - \lambda \), and the model collapses to:
It is noteworthy that model [15] looks similar to the traditional panel data stochastic frontiers model, except for one characteristic. Our model is dynamic as it involves a regression of \( y_{i,t} \) on \( y_{i,t-1} \). This model cannot be estimated, as is customary, by using a maximum likelihood estimator (MLE) because \( y_{i,t-1} \) is correlated with both \( \omega_{i,t} \) and \( u_i^+ \). Thus estimation of equation (15) by MLE gives us inconsistent estimates of the parameters. To avoid this endogeneity problem we might use the instrumental variable method if valid instruments for \( y_{i,t-1} \) are found.

Since it is unlikely that the time path of the excess-of-optimism term is the same for all firms in the market and finding good instruments is difficult in non-linear models like equation [15], we propose an estimation two-stage method that does not require making the above transformation and involves using MLE combined with a grid search over the \( \lambda \) parameter. In this case, equation [1] is estimated in the distributed lag form once we place expression [10] into [1]:

\[
y_{i,t} = a + b[z_{i,t+1}(\lambda) + u_{i,t}^+] + \epsilon_{i,t} \tag{16}
\]

Since \( z_{i,t+1} \) involves an infinitive series and we do not observe the infinitive past values of \( x_{i,t} \), we split \( z_{i,t+1} \) into two parts, one observed and the other not.

\[
z_{i,t+1}(\lambda) = \sum_{k=0}^{t-1}(1-\lambda)^kx_{i,t-k} + \sum_{k=t}^{\infty}(1-\lambda)^kx_{i,t-k} = z_{i,1t}(\lambda) + c\lambda^t \tag{17}
\]

Where

\[
c = \lambda^{-t}\left[\sum_{k=t}^{\infty}(1-\lambda)^kx_{i,t-k}\right] = \sum_{j=0}^{\infty}(1-\lambda)^jx_{i,t-j}, \quad j = k - t \tag{18}
\]

\( c \) is an unknown parameter to be estimated that can be interpreted as the expected profit for the first period. The equation to be estimated can be then written as:

\[
y_{i,t} = a + b[z_{i,1t}(\lambda) + c\lambda^t + u_{i,t}^+] + \epsilon_{i,t} \tag{19}
\]

or

\[
y_{i,t} = a + bz_{i,1t}(\lambda) + c'z_{i,2t}(\lambda) + bu_{i,t}^+ + \epsilon_{i,t} \tag{20}
\]

Where \( c' = bc \) and \( z_{i,2t}(\lambda) = \lambda^t \). We again use \( \lambda \) inside \( z_{i,1t} \) and \( z_{i,2t} \) as both depend on this parameter. It should be noted that for a given \( \lambda \) the equation [20] is a traditional stochastic

\[
y_{i,t} = \alpha + \beta x_{i,t} + \lambda y_{i,t-1} + \tau_{i,t} + b(1-\lambda) \cdot u_{i,t}^+
\]

This model can be estimated to identify firms with unsound expectations if \( \lambda < 1 \).
frontier model with two random terms and, hence, the other parameters of the model can be estimated, as is customary, by MLE techniques.

While assuming that $\varepsilon_{i,t}$ follows a normal distribution with zero mean and conventional variance $\sigma^2$, we need to choose a distribution for the asymmetric random term capturing the excess optimism, $u_{i,t}^+$, to estimate [20] by maximum likelihood. Although several simple distributions for the one-sided random term can be estimated, we choose the half-normal distribution for tractability reasons. The half-normal distribution, which is one of the most one-sided distributions employed in production frontier literature, is obtained from the truncation below zero of a random variable which follows a normal distribution with zero mean and variance $\sigma^2$. Skewness and truncation allow us to isolate the asymmetric random term capturing the excess optimism from other random shocks. The most important characteristic of the half-normal distribution is that the modal value of $u_{i,t}^+$ (i.e. the most frequent value) is close to zero, and higher values of $u_{i,t}^+$ are increasingly less likely (frequent). Therefore, the random term that captures the excess optimism is positively skewed, indicating that firms with unsustainable expectations are unusual and most of the firms have reasonable expectations about the future.

The marginal density function of $\omega_{i,t} = bu_{i,t}^+ + \varepsilon_{i,t}^+$ is given by

$$f(\omega_{i,t}) = \frac{2}{\sqrt{2\pi}\sigma} \left[1 - \Phi\left(-\frac{\omega_{i,t}}{\sigma}\right)\right] \cdot \exp\left\{-\frac{\omega_{i,t}^2}{2\sigma^2}\right\} = \frac{2}{\sigma} \phi\left(-\frac{\omega_{i,t}}{\sigma}\right) \Phi\left(\frac{\omega_{i,t}}{\sigma}\right)$$

Where $\sigma^2 = (b\sigma_u)^2 + \sigma^2$, $\rho = b\sigma_u/\sigma$, $\Phi(\cdot)$ and $\phi(\cdot)$ are the standard normal cumulative distribution and density functions respectively.\(^{14}\) As $\rho \rightarrow 0$ either $\sigma_u \rightarrow 0$ or $\sigma \rightarrow \infty$ and the symmetric error term dominates the one-sided error component in the determination of the composed error term, $\omega_{i,t}$. In this case the stochastic frontier model collapses to the single model introduced in the previous section with just a symmetric error term that can either be estimated by OLS or MLE.

From equation [21], we can obtain the log likelihood function for a sample of N firms observed over T periods:

$$lnLF = \frac{NT}{2} \cdot \ln(2/\pi) - NT \cdot \ln(\sigma) + \sum_{i=1}^{N} \sum_{t=1}^{T} \ln \left[\Phi\left(-\frac{\omega_{i,t}}{\sigma}\right)\right] - \frac{1}{2\sigma^2} \sum_{i=1}^{N} \sum_{t=1}^{T} \omega_{i,t}^2$$

\(^{14}\) See Stevenson (1980) and Kumbhakar and Lovell (2000, p. 140). Here, we have taken into account that the asymmetric random term capturing the excess optimism is multiplied by the parameter $b$ in equation (20).
where \( \omega_{i,t} = y_{i,t} - a - b z_{i,1t}(\lambda) - bc \cdot z_{i,2t}(\lambda) \). Assume that \( \lambda \) is known. For a given \( \lambda \), the ML estimator of the remaining parameters is the parameter vector that solves:

\[
(\hat{a}(\lambda), \hat{b}(\lambda), \hat{c}(\lambda), \hat{\sigma}(\lambda), \hat{\rho}(\lambda)) = \arg \max_{a,b,c,\sigma,\rho} \ln LF(a, b, c, \sigma, \rho | \lambda)
\]  

[23]

Next we can obtain the value of the likelihood function under the estimated parameters. Note that the ML estimator of \( (a, b, c, \sigma, \rho) \) is a function of \( \lambda \). Since the estimated parameters are functions of \( \lambda \), the value of the likelihood function is also a function of \( \lambda \), that is, \( \ln LF = \ln LF(\lambda) \). Since \( \lambda \) is unknown, it must be estimated from the dataset. We choose the value of \( \lambda \) for which \( LF(\lambda) \) is maximum, that is:

\[
\hat{\lambda} = \arg \max_{0 < \lambda < 1} \ln LF(\lambda)
\]  

[24]

This estimation strategy is the same as that mentioned in the previous section, except that we use MLE instead of OLS in the first-stage of the procedure. Both OLS and MLE are equivalent when the error term is made up of a single random variable; therefore, MLE or OLS yield the same parameter estimates. Since our error term in (15) is made up of two random variables and one of these variables is asymmetrically distributed, a MLE should be used.  

4 Dataset Description

The dataset used in this study came from a diverse range of sources. Information about capital and sales was collected directly from the annual reports. Both capital and sales were expressed in billions of dollars of 1970. Capital is a constructed variable that is equal to capital of previous period minus amortizations plus investments. The variable capital assigned to each year is the average of the beginning of the year and end of the year values.

---

15 If \( u_{i,t} = 0 \) and managers’ expectations are normal, the log likelihood function to be estimated is:

\[
\ln LF = -\frac{NT}{2} \ln (2\pi) - \frac{1}{2\sigma_e^2} \sum_{i=1}^{N} \sum_{t=1}^{T} e_{i,t}^2 - NT \cdot \ln(\sigma_e)
\]

This is the log likelihood function of a variable that follows a normal distribution. The resulting ML parameter estimates can be equally obtained in this case by using the method of least squares. As in Yéhou et al. (2010), the equation (20) can be written in a more compact form as \( y = \theta Z(\lambda) + \varepsilon \), where \( Z = (1, z_{i,1t}(\lambda), z_{i,2t}(\lambda)) \), and \( \theta = (a, b, c) \). The ordinary least squares estimator of \( \theta \) (as a function of \( \lambda \)) is given by

\[
\hat{\theta}(\lambda) = (Z(\lambda) Z(\lambda)^{-1} Z(\lambda)) y
\]

and the residual sum of squares is

\[
RSS(\lambda) = (y - \hat{\theta}(\lambda)' Z(\lambda)) (y - \hat{\theta}(\lambda)' Z(\lambda))
\]

The \( \lambda \) estimate can be defined as the value of \( \lambda \) with the minimum residuals sum of squares, that is, \( \hat{\lambda} = \arg \min_{0 < \lambda < 1} RSS(\lambda) \).
We studied five different firms (Walmart, Target, Kmart, Sears and May). Kmart declared bankruptcy in the year 2002 and merged with Sears in 2004. Therefore, we only include information about Kmart until 2002 and in the case of Sears until 2004. May acquired the company Associated Dry Goods in 1985. We decided to treat May as a different company after this event. Hence, we have six companies (Walmart, Target, Kmart, Sears, May pre-acquisition and May post-acquisition). We only have information about May until 2003.

We have collected control variables for improving the analysis such as the University of Michigan Consumer Sentiment Index and labor costs. Labor costs were calculated as general administrative expenses (SGAE) expressed in dollars of 1970 over the total number of employees. On the other hand, The Michigan index is based on 50 core questions about the general sentiment of American consumers about their personal finances, business conditions and buying conditions\textsuperscript{16}. It was generated for the first time in 1946 and the base period is 1966\textsuperscript{17,18}. We consider that these two variables influence capital investment decisions made by the discount chains. Tables 1 and 2 summarize the descriptive statistics for the dataset used in the paper. It is important to note that average capital growth is similar to the average growth of sales. More importantly, capital grows faster on average than the sales for every company.

5 Results

In this section we detail the steps that we followed to: (1) verify that OLS and MLE estimation provides the same results for the simplest scenario; (2) confirm the existence of a positive bias; (3) calculate the model with unrealistic optimism and (4) modify the original model by including additional variables that make our estimations more robust.

The first step requires the estimation of the expression [9]. The grid search over the parameter lambda is done over 396 possibilities (from lambda equals to 0.0125 to 0.9975 in increments of 0.0025). The calculations were done using sales as an independent variable and capital as a dependent variable. We performed the grid search using the OLS technique and the MLE technique like in equation [23] under the premise that $\rho \to 0$. The results are shown in figure 1. It is important to note that the residual sum of squares reaches its minimum exactly


\textsuperscript{17} Ibid.

\textsuperscript{18} Other variables were tested, but not included in the final version of the theses due to the impossibility of reaching convergence. These variables were Housing Price Index (as a proxy for Retailing Space Price Index), and consumer credit.
when the log likelihood function is the maximum. Lambda is equal to 0.81. Table 3 presents the results for the OLS estimation when the RSS reach the minimum and Table 4 shows the coefficients using MLE technique.

The OLS residuals allow us to perform a test on the existence of a positive u. Kumbhakar and Lovell (2000) list two types of tests where the null hypothesis is that u=0. The first test was developed by Schmidt and Lin (1984) based on the second and third moments of the OLS residuals. Nevertheless, the distribution of this test is not widely published (Kumbhakar and Lovell, 2000, p. 73). The other test was developed by Coelli (1995) and it is asymptotically normally distributed with mean zero and variance equal to one:

\[ z \sim \sqrt{\frac{m_3}{6m_2^2/1}} \tag{[25]} \]

Where \( m_3 \) and \( m_2 \) are the third and second moments of the OLS residuals, and I is the number of observations. For our estimation, the test yielded 16.04. This means that the residuals are positively skewed (as expected) and that u is different from zero with a 0.01% of significance. However, these tests are based on asymptotic theory (Kumbhakar and Lovell, 2000, p.73). Therefore, the test result is good, but it is not conclusive.

The next step is the calculation of equation [20] in the simplest form possible. After performing the grid search we found that the lambda that minimizes the log likelihood function is 0.795, smaller than in the standard case. Table 3 shows the coefficients for equation [20]. In this scenario \( a \approx -0.9653; b \approx 0.3657 \) and \( c' \approx 0.6293 \). All the coefficients were significant. This outcome implies that if the predicted sales volume increases by 1 billion, total capital would increase by 365 million approximately. The log-likelihood ratio test rejects the null hypothesis that u is equal to zero at 0.01 significance level. We call these results “model 1”.

Now we can make an estimation of the level of optimism for each of the five firms. Figure 2 reflects the calculations for the simplest case. The results show higher levels for Target. Walmart, the company with the best performance in terms of sales volume, has a moderate level of optimism and Kmart has the lowest level of optimism. The value of \( u \), which measures optimism, is very large in most of the cases and it seems to increase with time. We try to correct this by adding a trend.

The coefficients with the trend are in table 5. All of them are significant and very close to those reported in the previous regression. The trend has a positive influence on capital
acquisition. Figure 3 shows that the reported optimistic levels are much more moderate although they are still high.

The final step is to include some control variables besides the trend. Equation [20] is modified as follows:

\[ y_{lt} = a + bz_{i,lt}(\lambda) + c'z_{i,2t}(\lambda) + \theta \text{Trend} + \theta x_{1t} + \ldots + \theta x_{rt} + bu_{lt}^t + \varepsilon_{lt} \quad [21] \]

Where \( x_{rt} \) is a control variable and \( r=1,\ldots, R \) represent the number of variables analyzed. We test whether control variables make a difference with respect to our findings in the simplest model. We have two additional models. The third model includes the University of Michigan Consumer Sentiment Index and the fourth model includes labor costs. Table (5) reveals that the coefficients for \( z1, z2 \) and trend are stable and significant. An increment of one billion in expected sales, increases future capital by more than 300 million. Every year capital investment increases by approximately 6 million.

The influence of consumer sentiment captured by the Michigan index is negative. This might seem paradoxical. If consumers are more confident about the future, managers choose lower capital levels. An explanation could come from the nature of the discount retailing business. Some of these businesses thrive during bad times (e.g. Basker, 2008 finds that Walmart sells “inferior goods” in the economic sense, increasing its revenues during economic downturn). Therefore, if consumers have a negative sentiment about the future, it might be an opportunity to increase their clientele. Labor costs also have a negative effect on capital investment. It seems coherent that if labor costs per worker are increasing the company has less money to invest in capital.

The values for sigma \( \nu \) and \( u \) are positive. The null hypothesis of the LR test \( u=0 \) was rejected with 1% significance in the first two models and 5% and 10% in the last two models. Lambda did not fluctuate much. It was between 0.75 and 0.795. If lambda is equal to zero, then the expected volume of sales is equal to the previous one plus the bias term. Conversely, if lambda is equal to one then the expected sales volume is equal to the previous prediction plus a difference among the biases of two consecutive periods. Therefore, if lambda is close to one it means that the prediction error is not taken into consideration when expectations are formed. The outcome reveals that managers usually correct their estimations only taking into consideration 20% to 25% of the previous mistake.
Figures 3 to 5 represent graphically the optimistic levels derived from models 2 to 4. We found that the results are very similar. Target is the company with the highest level of optimism and Kmart has the lowest. Walmart and Sears have moderate levels of optimism. May’s decision to acquire a new company had a negative effect on the levels of optimism reported. Before the acquisition May had the highest levels of optimism. After the acquisition, May’s levels of optimism dropped substantially.

From these results, we cannot conclude that high levels of optimism are related with business failure. Kmart has the lowest levels of optimism of the five companies. With the exception of 1984, Kmart’s reported levels of optimism were almost flat. Sears reported diminishing levels of optimism as its market share shrank. May’s post-acquisition drop might signal an adjustment period after a merge. On the other hand, Walmart’s optimism decreased with time and their reported performance levels are moderate. Target and Walmart’s results support the idea that optimism is related with high performance. Nevertheless, our results are far from conclusive.

6 Conclusions

In this study we presented a new application of the stochastic frontier literature. We apply this methodology to assess the level of optimism interpreting the previous technical inefficiency as excess optimism. The stochastic frontier estimation had an additional level of difficulty since it was dynamic which could require the use of instrumental variables. We selected an alternative approach by using a grid search over the parameter lambda.

Our results corroborate partially with our expectations. First, it has been proved that under the assumption of no bias, OLS estimation and MLE estimation yield the same results. We performed a test with the OLS residuals to verify whether or not unrealistic optimism exists and the result confirmed this assumption. We consider this a partial confirmation since the test relies on asymptotic theory. The next step was to estimate the model with the positive bias. The log likelihood test rejected the null hypothesis that the bias term was different from zero. However, when this $u$ term was calculated, the outcome reveals very high levels of excess optimism. The final step was to incorporate additional control variables like a trend, the index of consumer sentiment and labor costs into the model. The outcome did not modify our previous assessment much. The new results show that in general the companies that perform poorer such as Kmart, exhibit low levels of optimism while other firms such as
Walmart or Target present high levels of optimism. Our results challenge the idea of rational expectations; managers make systematic mistakes in their assessments of future performance.

7 References


Koyck, L.M. (1954), Distributed Lags and Investment Analysis, North Holland, Amsterdam


8 Figures & Tables

Figure 1
MLE & OLS Estimation First Model (Equation 9)
Using sales as an independent variable

![MLE & OLS Estimation First Model](image)

Figure 2
Optimism Level for the Five Selected Firms.

![Optimism Level for the Five Selected Firms](image)
Figure 3
Model 2; Adding a Trend Variable

Figure 4
Model 3; Adding Michigan Index
Figure 5
Model 4; Adding Labor Costs
Table 1
Description of the Variables Used

<table>
<thead>
<tr>
<th>Variable</th>
<th>Source</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>Annual reports of the studied companies. Calculated from the Balance Sheet.</td>
<td>Amounts expressed in billions of dollars of 1970.</td>
</tr>
<tr>
<td>Sales</td>
<td>Annual reports of the studied companies. Calculated from the Income statement.</td>
<td>Amounts expressed in billions of dollars of 1970.</td>
</tr>
<tr>
<td>Michigan Index of Consumer Sentiment</td>
<td>Thompson Reuters/University of Michigan <a href="http://www.sca.isr.umich.edu/main.php">http://www.sca.isr.umich.edu/main.php</a></td>
<td>It is based on a survey conducted by the University of Michigan since 1946. The survey has 50 core questions and it is conducted telephonically. 500 people are interviewed. The base period is 1966. Calculated as the ratio of selling, general and administrative expenses (SGAE) to the total number of employees.</td>
</tr>
<tr>
<td>Labor costs</td>
<td>Annual reports of the studied companies.</td>
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</table>

Table 2
Descriptive Statistics

<table>
<thead>
<tr>
<th>Sales (Millions 1970)</th>
<th>Average</th>
<th>Std. Dev/Avg</th>
<th>Geo. Avg. Growth</th>
<th># obs</th>
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<tbody>
<tr>
<td>Walmart</td>
<td>20,369.64</td>
<td>114%</td>
<td>20.17%</td>
<td>37</td>
</tr>
<tr>
<td>Target</td>
<td>4,944.42</td>
<td>65%</td>
<td>6.70%</td>
<td>37</td>
</tr>
<tr>
<td>Kmart</td>
<td>6,269.46</td>
<td>52%</td>
<td>2.71%</td>
<td>31</td>
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<tr>
<td>Sears</td>
<td>7,851.81</td>
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<td>-0.86%</td>
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<tr>
<td>May Pre-merge</td>
<td>402.83</td>
<td>130%</td>
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</tr>
<tr>
<td>May Post-merge</td>
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<td>-1.59%</td>
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<tr>
<td>Total general</td>
<td>6,806.00</td>
<td>172%</td>
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</table>

<table>
<thead>
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<th></th>
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<tbody>
<tr>
<td>Walmart</td>
<td>5,169.54</td>
<td>129%</td>
<td>23.86%</td>
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</tr>
<tr>
<td>Target</td>
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<td>9.11%</td>
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<td>May Post-merge</td>
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<td>Total general</td>
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<td>165%</td>
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</table>

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<tr>
<th></th>
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<tbody>
<tr>
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<td>5.40</td>
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<tr>
<td>Target</td>
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<tr>
<td>Kmart</td>
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<td>10.77%</td>
<td>0.97%</td>
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</tr>
<tr>
<td>Sears</td>
<td>7.57</td>
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<td>Total general</td>
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<tr>
<td>Michigan Index of Consumer Sentiment</td>
<td>86.77</td>
<td>13.25%</td>
<td>-0.06%</td>
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26
### Table 3

**OLS Estimation of the Model with No Excess Optimism**

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<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
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<td>2</td>
<td>102030000</td>
<td>F( 2, 166)</td>
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<tr>
<td>Residual</td>
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<td>166</td>
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<tr>
<td>Total</td>
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<td>168</td>
<td>12596101.5</td>
<td>R-squared</td>
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<table>
<thead>
<tr>
<th>Coef.</th>
<th>Std. Err.</th>
<th>t</th>
<th>P&gt;t</th>
<th>[95% Conf. Interval]</th>
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<tr>
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<td>59.16</td>
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### Table 4

**MLE Estimation of the Model with No Excess Optimism**

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<thead>
<tr>
<th>Coef.</th>
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<table>
<thead>
<tr>
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<th>[95% Conf. Interval]</th>
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<td>Z2</td>
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<td>0.2109</td>
<td>0.0080</td>
</tr>
<tr>
<td></td>
<td>***</td>
<td>***</td>
<td>***</td>
<td>**</td>
</tr>
</tbody>
</table>

**Table 5**

Results of Modified Version