Effects of the heterogeneity of the variances on reliability generalization: An example with the Beck Depression Inventory

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The studies of Reliability Generalization (RG) analyze estimates of the reliability of scores from a test provided by a set of studies. As their goals and designs are usually very varied, the sampling of individuals obeys very different schemas. Thus, the variances of the scores might be more heterogeneous than expected from random sampling. Two main problems associated with this potential source of heterogeneity should be taken into account. First, heterogeneity has been usually identified subjectively, not very rigorously. Second, once identified, it has not been taken into account in subsequent analyses. In previous papers, various ways to face both problems have been proposed. The procedures are summarized and applied to a set of 65 independent studies that report estimates of the internal consistency of the Beck Depression Inventory. The results show why any study of RG should take into account the heterogeneity of the variances. In addition to this, the only source that additionally accounts for significant variance in the coefficients is the version of the test employed: the second and third versions of the test involve significant increases in the internal consistency. The consequences of ignoring the heterogeneity of the variances are discussed.

Efectos de la heterogeneidad de las varianzas en la generalización de la fiabilidad: un ejemplo con el Inventario de Depresión de Beck. Los estudios de Generalización de la Fiabilidad (GF) analizan estimaciones de la fiabilidad de las puntuaciones de un test proporcionadas por un conjunto de estudios. Como normalmente sus objetivos y diseños son extremadamente variados, sus muestras de individuos obedecen a esquemas muy diferentes. Consecuentemente, las varianzas de las puntuaciones serán más heterogéneas de lo esperado por mera fluctuación aleatoria. Hay dos problemas principales asociados a esta fuente potencial de heterogeneidad que se deberían tener presentes. Primero, la heterogeneidad ha sido normalmente identificada subjetivamente, de forma poco rigurosa. Segundo, una vez identificada no se ha tenido en cuenta en los análisis posteriores. En trabajos previos se han propuesto formas de afrontar ambos problemas. Se resumen esos procedimientos y se aplican a un conjunto de 65 estudios independientes que aportan estimaciones de la consistencia interna del Inventario de Depresión de Beck. Los resultados muestran por qué cualquier estudio de GF debería tener en cuenta la heterogeneidad de las varianzas. Aparte de ésta, la única fuente que explica adicionalmente una varianza significativa en los coeficientes es la versión: la segunda y tercera versiones del test implican incrementos significativos de la consistencia. Se discuten las consecuencias de ignorar la heterogeneidad de las varianzas.
fluctuations) in the samples’ means and variances should be interpreted as an indicator of the presence of varied sampling schemes in the studies included in the meta-analysis.

Once the presence of heterogeneity of variances has been diagnosed, the role of other moderator variables can be analyzed, incorporating them as additional moderator variables to the variances. That is, instead of analyzing them isolated, they are added to the variances in models with multiple moderators. Specifically, Botella and Suero (in press) have argued that the formula that allows obtaining, under range restriction conditions, the reliability of a test from the reliability of the original test and its variance (Crocker & Algina, 1986; Feldt & Brennan, 1989; Lord & Novick, 1968) provides values quite approximate when applied to the coefficients of internal consistency. Previous analyses suggest that an appropriate way to analyze the role of potential moderating variables is developing a base model to predict the reliability, with the sample variance as the only predictor. Subsequently, other predictors are added to assess any significant increase in the variance accounted for the model.

Botella and Suero (in press) have also highlighted the plausibility of the linear model of equation 1 (see technical details in the appendix):

\[
\log(1-p) = \log(\sigma_j) - \log(\sigma_f)
\]

Model (1) has two main advantages. The first one is that it is more credible. Specifically, previous attempts to incorporate samples’ variances have assumed, rather arbitrarily, a linear relationship between the reliability coefficients and the variances. On the contrary, the linear relationship between the logarithms conveyed in equation (1) is directly derived from the assumptions of the Classical Test Theory. The second advantage is that the variance of \(\log(1-p)\) is known (Bonett, 2002). Additional moderator variables can be incorporated into this predictive model.

The purpose of the present paper is to show how this procedure can have a significant effect on the conclusions of reliability generalization studies that employ the Cronbach’s alpha coefficient as the effect size for reflecting the internal consistency of the scores obtained with a test. Specifically, we show how it is applied to a set of 65 studies in which the estimates of Cronbach’s alpha index of internal consistency (Cronbach, 1951) obtained with the Beck Depression Inventory are reported (BDI, Beck, Ward, Mendelson, Mock, & Erbaugh, 1961). We chose this test because it is well known and some RG studies with it have been already published (e.g., Yin & Fan, 2000).

**The BDI**

The Beck Depression Inventory (BDI) is one of the most widely used instruments to assess the severity of depressive symptoms, in both clinical and nonclinical populations. It includes a somatic or physical subscale and a psychological or affective subscale.

The somatic or psychological subscale includes a variety of elements (e.g., loss of pleasure, crying, loss of energy, and so on). The affective subscale includes several elements (e.g., pessimism, past failures, guilt feelings, and others) (Beck et al., 1961). The tool consists of 21 items in a 4-point scale (0-3), so that scores range from 0 to 63 (higher total scores indicate more severe depressive symptoms).

Several versions of the test have been proposed. The Beck Depression Inventory (BDI, Beck et al., 1961) was the original version. The respondents are instructed to select the statement that seems to fit him/her the best at the present time. In some items, two alternative statements are presented and are assigned the same weight (increase and decrease of sleep and appetite).

This version was revised in 1978 (BDI-IA; see Beck, Rush, Shaw, & Emery, 1979). In this version the two alternative statements of some items were removed. Furthermore, the respondents are asked to rate how they had been feeling during the preceding two weeks.

The BDI-II (Beck, Steer, & Brown, 1996) replaced the BDI and the BDI-IA; it is based on the Diagnostic and Statistical Manual of Mental Disorders, Fourth Edition (DSM IV) of the American Psychiatric Association. It also includes 21 items in a 4-point scale. Items related to body image, work difficulty, weight loss, and somatic preoccupation, were replaced with items related to agitation, concentration difficulty, and loss of energy. Additionally, the instructions remained as in the previous version (respondents are asked to rate how they have been feeling for the past two weeks). According to Beck et al (1996), the BDI-II has been found to demonstrate high internal consistency, adequate validity and diagnostic discrimination.

The above are the three versions employed in the studies of the present meta-analysis (table 1). There are other shortened versions, as the Beck Depression Inventory for Primary Care (BDI-PC), also known as a BDI fast screen (BDI-FS), but we have not included studies that estimate the internal consistency with this version.

**Method**

**Study selection**

We included 65 studies in which an empirical estimation of the internal consistency of the scores in the BDI is reported. Most of them were also integrated in the study of Yin and Fan (2000), although not all the studies used in their paper are included, because of restricted accessibility or lack of statistical details (e.g., mean and variance of the scores). However, we do not pretend to be exhaustive neither in the inclusion of studies nor in the conclusions. The goal of our study is to give an example of the methodological advantages of the framework for RG analysis proposed by Botella and his colleagues (Botella & Suero, in press; Botella et al., 2010). The studies included in the analysis are marked with an asterisk in the reference section.

**Codification of studies**

Each study was coded according to several moderator variables, both contextual and methodological (Botella & Gambara, 2002). Several outcome variables were also collected, specially the Cronbach’s alpha coefficient and the means and variances of the total scores. Table 1 shows the main variables recorded and summarizes the descriptive statistics for the coded variables.

**Statistical analyses**

Although the effect size index is the alpha coefficient itself, it was previously transformed to \(\log(1-\alpha)\). The variance of this transformation (Bonett, 2002) is approximately equal to \(2J/(J-1)\) \((N-2)\), where \(J\) is the number of items and \(N\) is the number of participants. The statistical analyses have been done with SPSS, including the macros provided by Lipsey and Wilson (2001). The
specific component of inter-studies variance has been estimated via the method of moments. The studies have been weighted by 1/S^2, the inverse of its variance, although here is equivalent to weighting the method of moments. The studies have been weighted by 1/S^2, specific random fluctuations from a fixed effects model. However, in further analysis we have always used random effects models (Hedges & Vevea, 1998).

**Heterogeneity of the variances**

As explained above, if the estimates included in a RG study involve very different sampling schemes, the samples’ variances of the observed scores would be very heterogeneous. In this particular case it is expected to be so, given the variety of designs and the compositions of the samples in the primary studies. Let’s see some examples:

- In the study of Sun, Hui and Watkins (2006) two extreme groups were selected according to their scores on dysphoria, but the internal consistency was estimated including all participants together. The inclusion of extreme samples in a trait strongly associated with depression tends to significantly increase the variance, as there are few scores from the central part of the population’s distribution.
- The work of Dozois, Dobson and Ahnberg (1998) is a psychometric analysis of the test, conducted with a sample of psychology undergraduates.
- Weeks and Heimberg (2005) obtained the internal consistency in an application to adult patients with Generalized Anxiety Disorder. It is sampled selectively from high values of anxiety, a variable with a well-known positive correlation with depression.
- Grothe, Dutton, Jones, Bodenlos, Ancona and Brantley (2005) used the BDI to study a sample of low income and African American medical patients. We do not know in what way the patient selection criteria in this study represent a biased selection from the general population.

Obviously, we should not expect similar scores’ variances in those samples. This heterogeneity will generate a greater heterogeneity in the coefficients of reliability than that resultant from a random sampling of the general population in all the primary studies.

The framework we have proposed (Botella & Suero, in press; Botella, Suero, & Gambia, 2010) consists of (a) identify the presence of variations in the sampling schemes employed; (b) incorporate the variation due to these schemes in the explanatory models; and (c) include other moderator variables to assess if they add significant explanatory capacity to that already provided by the sampling schemes.

Let see each of these steps. To identify the presence of a variety of sampling schemes, Botella, Suero and Gamba (2010; Botella & Suero, in press) have proposed analyzing the degree of heterogeneity of the samples’ means and variances. In our case, the samples’ means show a significant heterogeneity [Q(64)=10575.9; p<.0001]; this test is enough to conclude about the presence of a varied sampling schemes. However, we have also analyzed the variances’ heterogeneity. The result is convergent, as the variance also shows a significant heterogeneity [Q(64)=1907.5; p<.0001]. However, as this test is vulnerable to violations of its assumptions, given that samples’ variances are asymmetrically distributed (chi-square with N-1 degrees of freedom), we have reanalyzed this heterogeneity for only the studies with N>100, so that their distribution can be considered approximately normal. The results again lead us to the conclusion that the heterogeneity is larger than expected from random fluctuations [Q(54)=1838.2; p<.0001]. The conclusion is clear: the studies have been carried out under different sampling schemes, and this is certainly a source of variation in the alpha coefficients collected from the studies.

**Building a model**

The next step consists in the development of a base-model for the variability of the coefficients, using only the studies’ variances as an explanatory variable. The weighted least squares fit of the model expressed in equation 1 gives us the result that appears in equation 2, where the only predictor is the sample variance, and Y represents the prediction made by the model for the Log(1-α), as expressed in Equation 1. The model explains a significant proportion of the variance in the coefficients [Q(1)=38.357; p<.0001; specific variance component of the model, ν=0.05125; R^2= .348].

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
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<tbody>
<tr>
<td><strong>Main variables and categories coded or calculated from the primary studies</strong></td>
</tr>
<tr>
<td>Version of the questionnaire</td>
</tr>
<tr>
<td>BDI, original version of 1961</td>
</tr>
<tr>
<td>BDI-IA, version of 1978</td>
</tr>
<tr>
<td>BDI II, revised version of 1996</td>
</tr>
<tr>
<td>Language</td>
</tr>
<tr>
<td>English</td>
</tr>
<tr>
<td>Spanish</td>
</tr>
<tr>
<td>Others</td>
</tr>
<tr>
<td>Type of sample</td>
</tr>
<tr>
<td>Students</td>
</tr>
<tr>
<td>Patients (inpatients and outpatients)</td>
</tr>
<tr>
<td>General population</td>
</tr>
<tr>
<td>Mixed</td>
</tr>
</tbody>
</table>

| Type of patients | | |
| Affective disorders | 9 | (26.5) |
| Others/unspecified | 18 | (52.9) |
| Various (groups with different diagnosis) | 7 | (20.6) |

<table>
<thead>
<tr>
<th>Mean (St. dev)</th>
<th>Min.-max.</th>
</tr>
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<tbody>
<tr>
<td>Sample size (N)</td>
<td>359 (382)</td>
</tr>
<tr>
<td>Mean age of the sample</td>
<td>33 (14.1)</td>
</tr>
<tr>
<td>Sex (% of women)</td>
<td>62.9 (22.8)</td>
</tr>
<tr>
<td>Mean of the total scores</td>
<td>14.0 (6.7)</td>
</tr>
<tr>
<td>Variance of the total scores</td>
<td>87.5 (47.2)</td>
</tr>
<tr>
<td>Cronbach’s alpha</td>
<td>.881 (.038)</td>
</tr>
</tbody>
</table>
Given the conclusion reached about the sampling schemes, it should not be accepted for this set of coefficients any model that does not include the sampling variance as an explanatory variable. Other models below are developed by including additional moderators to this base-model. Only if the percentage of variance accounted for the model increases significantly, it is acceptable.

We tested a high number of models, arising from the combination of predictors. Our final model (a mixed effects model) includes only the test version, besides the sample variances (the weighted average reliability is .827, .873 and .901 for the studies employing the three versions of the test, respectively). The model is specified in equation 3; the versions of the test are coded with ordered values (1, 2, 3, for the first, second, and third versions, respectively). Naturally, it shows a significant result [Q1(1)= 88.918; \( p<0.001 \)]; specific variance component of the model, \( \psi = 0.03202 \); the proportion of variance accounted for the model rises to \( R^2 = 0.525 \).

\[
Y' = -0.422 - 0.284 \cdot \log(S) - 0.213 \cdot V_i
\]  

We have tested whether the increase in the explained variance is significant using a statistic for nested models (Judd & McClelland, 1989; Maxwell & Delaney, 1990). The result is significant \( F(1,62)=36.53; p<0.001 \), so that we can conclude that the inclusion of the version increases significantly the explained variability. Furthermore, the tests for models that include other additional moderators have not shown significant increments in the explanatory power.

In summary, the most parsimonious model is one that includes two predictors. The first one is the samples’ variances, which we had already decided should be included because the homogeneity tests of the means and variances have shown statistically significant. The second one is the version of the test, distinguishing between the 1961 (BDI), the 1978 (BDI-IA), and the 1996 (BDI-II) versions. The model establishes that the Cronbach’s alpha increases in the BDI-IA version, as compared to the first version. Specifically, for a sample with variance equal to 75 the predicted alpha increases from .845 to .875; in a similar vein, it increases from .875 to .900 if the BDI-II (third version) is employed instead of the BDI-IA (second version).

Consequences of ignoring the heterogeneity of the variances

Although some previous studies of RG have pointed out a significant association between the reliability coefficients and the samples’ variances, it rarely has been taking into account for further analysis. Let’s see where this approach would have taken us.

We have fit models with several moderators, and the main significant outcomes are those related to categorical models, as Sample type and Language, and also to the continuous model created with Sex. Regarding to the moderator Sample type, it explains a significant part of the variance \( Q_f(3)= 9.158; p<0.03 \); the participants selected from the general population show a higher internal consistency (.900), then general patients (.896), and finally the group with «various» (different groups) and the students (.874 and .873, respectively). Language also shows a significant result when the category ‘English’ (the language of the original version) is compared to the category ‘other languages’ \( Q_f(1)= 4.710; p<0.04 \), although the decrease is small; the studies with the English version have associated an average coefficient of .891, whereas it is reduced to .873 in the studies with other groups.

Sex (percentage of women) also shows a significant result \( Q_f(1)= 5.542; p<0.02 \). As the negative slope indicates, a higher percentage of women is associated with a lower internal consistency; however the percentage of explained variance is rather small \( R^2 = 0.075 \).

An analysis that ignores the samples’ variances lead to conclude that all of the above moderators are relevant to predict (or explain) variations in the coefficients. However, these variables also show significant associations with the samples’ variances. The variance heterogeneity explains in a more parsimonious way the apparent association among them and the internal consistency.

Discussion

When in a meta-analysis of RG the variations in the sampling schemes of the studies’ samples are ignored, the conclusions can be severely misguided. The presence of varied sampling schemes must be diagnosed by testing for significant heterogeneity on samples’ means and variances, not in a subjective way. This has been shown in a practical example with the BDI. The results support the ideas developed in our previous work (Botella & Suero, in press; Botella, Suero, & Gambara, 2010).

Once decided the presence of various sampling schemes, the subsequent analysis should incorporate the samples’ variances as an explanatory variable. Any other moderator variable should be added to it, not replace it; otherwise, it could be capitalizing a relationship between the moderator variable and specific sampling schemes.

We have applied this analysis framework to 65 estimates of the internal consistency (Cronbach’s alpha) of the Beck Depression Inventory (BDI), concluding that the most parsimonious model to explain the variations in the coefficients is one that includes as predictors (or moderators) the samples’ variances and the test version.

These findings mean that the observed variability in the internal consistency coefficients does not imply a differential functioning of the test according to the sample characteristics, such as the patient status, the average age, sex, or other. The main source of variation is associated to the sampling of true scores (in the sense of the Classical Test Theory; Lord & Novick, 1968). According to a well known relationship in psychometrics, the more heterogeneous is the sample, the larger is the coefficient. But this does not mean that the psychometric quality of the individual scores is different. The alpha coefficient is a group statistic and its value depends on a circumstantial characteristic, the heterogeneity of the sample, which has nothing to do with the measurement, but much with the decisions made for the sampling of the participants.

The second source of variation in the data is the test version. Successive versions of the BDI (1961, 1978 & 1996) have been developed with the aim of improving the psychometric quality of the test. Our findings support that this goal has been accomplished in each new version, or at least in the three versions we have included in our study. Each new version has conveyed an significant increment in the internal consistency of the scores.

In general, in the field of meta-analysis are often welcome the studies that show that there are moderator variables associated with effect sizes of different magnitude. These refinements improve our understanding of the phenomena under study, and point out the lines for future research (Borenstein, Hedges, Higgins, & Rothstein,
Following the suggestion of one of the reviewers we have reanalyzed the data including the versions of the test as two fictitious coding variables. In the first one the value 1 is associated to version 2, whereas in the second one it is associated to version 3. The main consequence of this alternative way to manage the version is that it is lost one degree of freedom when the model is compared with the base model. However, the conclusion does not change, as the statistic is still statistically significant \( F(2,61) = 18.08; p < .001 \).

Acknowledgements

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Appendix

Classical Test Theory assumes that the observed score \( (X) \) can be defined as the true score \( (T) \) plus some random error \( (E) \), and several distributional assumptions (Crocker & Algina, 1986). The reliability is defined as,

\[
\rho = \frac{\sigma_T^2}{\sigma_X^2} \tag{A1}
\]

but the assumptions allow express it also as,

\[
\rho = 1 - \frac{\sigma_E^2}{\sigma_X^2} \tag{A2}
\]

Under range restriction conditions the variance is different \( (\sigma_T^2) \), but as the variance of the errors is constant, the reliability under those conditions \( (\sigma') \) is,

\[
\rho' = 1 - \frac{\sigma_E^2}{\sigma_X^2} \tag{A3}
\]

When the studies included in a RG meta-analysis involve varied sampling schemes that generate different variances, the expected reliability for each one can be obtained from (A3); this equation can be expressed as,

\[
1 - \rho' = \frac{\sigma_E^2}{\sigma_X^2} \tag{A4}
\]

Taking logarithms we reach equation (1),

\[
\log(1-\rho') = \log(\sigma_E^2) - \log(\sigma_X^2)
\]

References

(A references marked with an asterisk indicate studies included in the meta-analysis).


