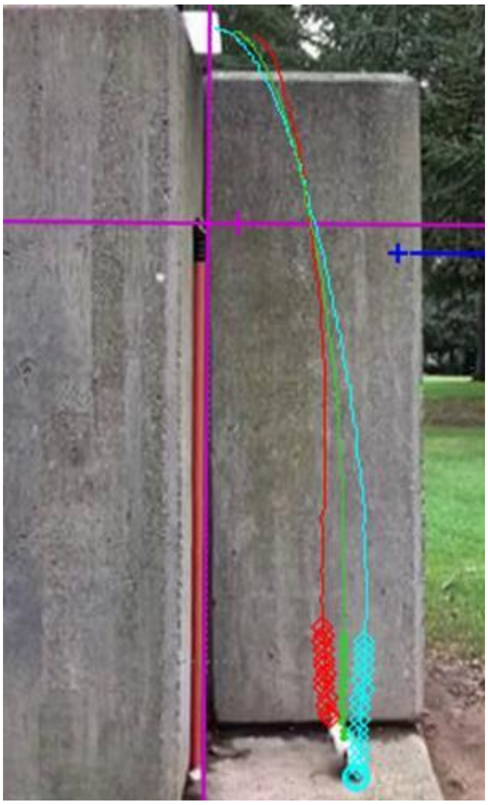
Murphy Law with a falling toast



Summary

The motion of a toast falling under the action of gravity and rotating about its center of mass is a typical example of Murphy's law: "if a toast falls from a table it will always land on the floor with its butter face on the carpet".

This "law" can be experimentally tested with a video of a falling flat object. In the first Tracker file, the motion of the toast end points and its Center of Mass is analyzed as it falls from a high table. The center of mass follows an approximately parabolic motion with a certain horizontal friction. In the second file the rotation of the toast end points relative to the Center of Mass is displayed.

The experiment clearly shows that the toast makes a half turn during its falling trajectory so that it arrives upside down to the floor.

Equations of motion

The center of mass of a falling object solely under the action of weight follows a typical parabolic trajectory **r**(t) = (x, y)

$$x = x_0 + v_{0x}t$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

and velocity **v**(t) = (v_x, v_y)

$$v_x = v_{0x}$$

$$v_y = v_{0y} - gt$$

The experimental data for the Center of Mass of the falling toast follow closely a parabolic motion during the first 0.3 seconds of motion. Then, a drag force sets in reducing horizontal velocity to zero and, at t=0.4s, vertical velocity deviates below the expected linear trend indicating the onset of certain vertical drag.

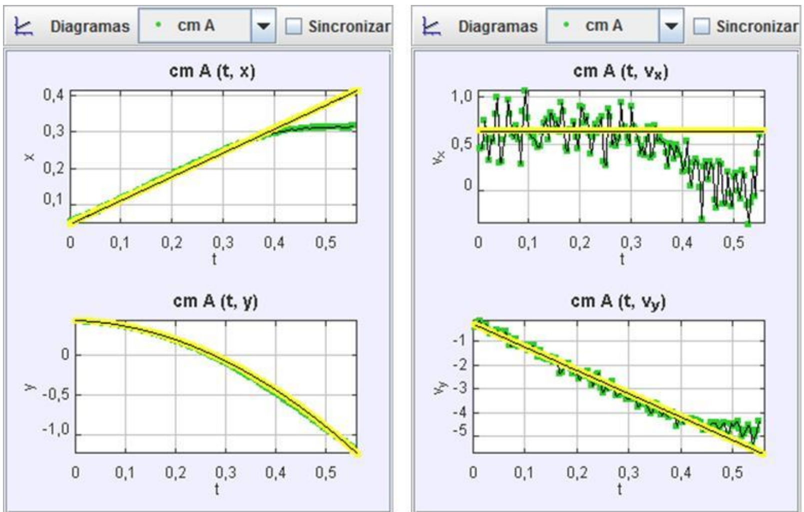


Figure 1. Center of mass motion of a falling toast: green points, experiment; yellow points, parabolic model.

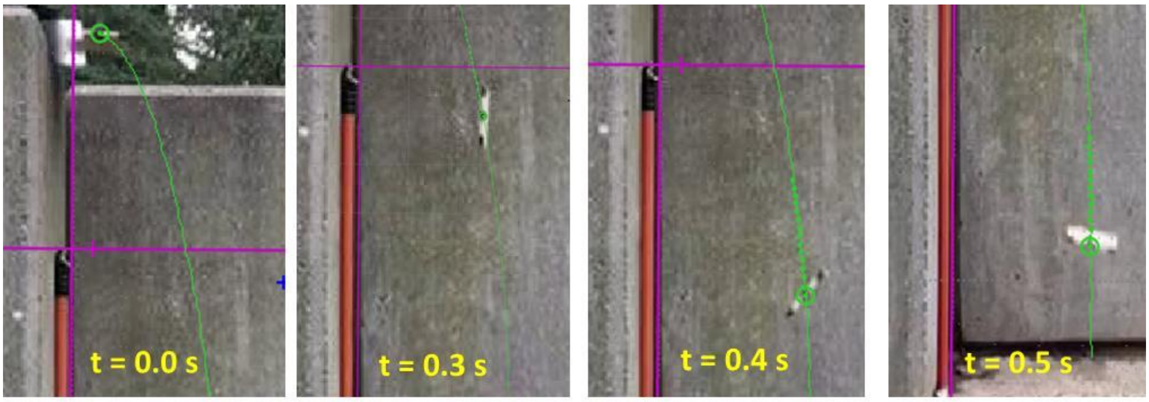


Figure 2. Selected frames of during falling toast trajectory. At t=0, it is horizontal; at t=0.3 s it is vertical; at t=0.4 s it is oblique by about 45º; at t = 0.5 s, it approaches the floor up-side down.

The onset of drag can be correlated with the different orientation of the toast as it falls down from an initial horizontal position at t=0, with its "butter face" up (see Fig. 2). At t = 0.3 s, when horizontal drag sets in, the toast is almost vertical (maximum horizontal drag, minimum vertical drag). Then, at t=0.4 s, when vertical drag starts to become relevant, the toast has rotated by 45º, increasing its vertical section. Finally, at t = 0.5 s, the toast has fully rotated up-side down becoming horizontal but with its "butter face" down.

Figure 3, shows the motion of the toast ends (pink and light blue), using the Center of Mass as a reference frame. Both ends follow a similar circular trajectory, as expected from rigid body rotation. Angular speed is almost uniform at 350º/s ≈ 6 rad/s, i.e. with rotation period T ≈ 1s.

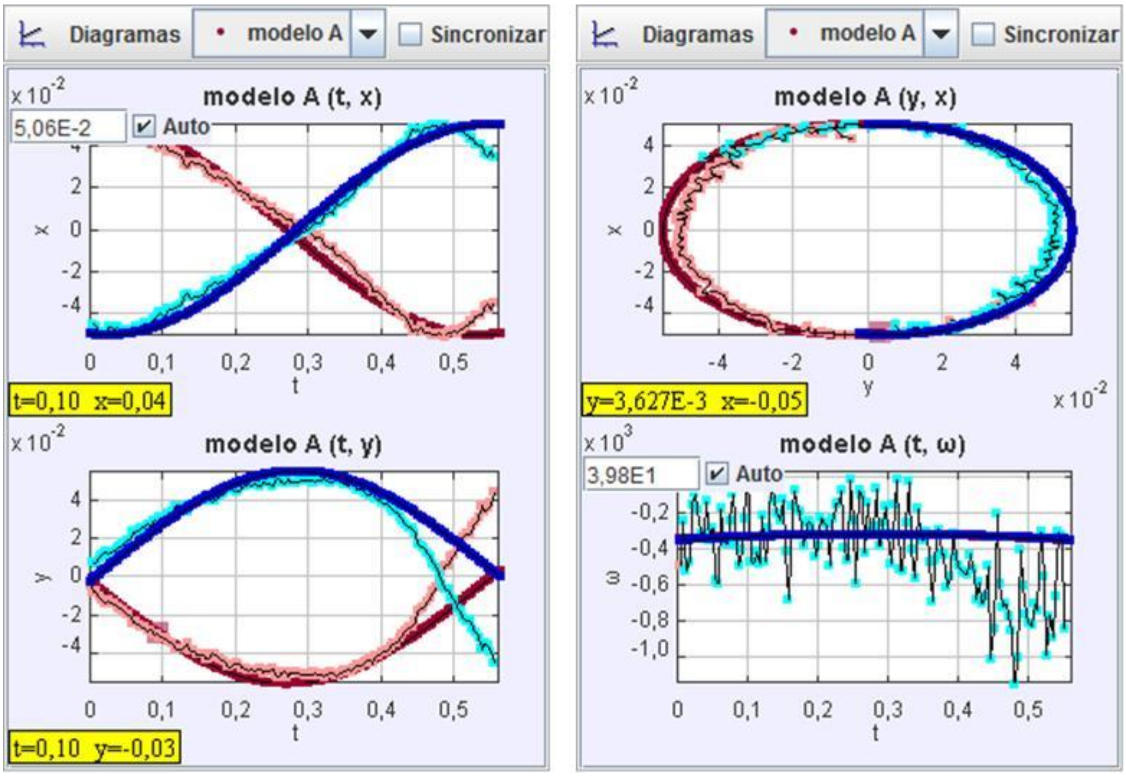


Figure 3. Motion of the toast end points (pink and light blue) in the Center of Mass reference frame. Dark Blue and Red symbols correspond to a uniform rotation model with w = 350º/s ≈ 6 rad/s.

That is, during half a period $T/2 = 0.5$ s, as the toast turns upside down, the center of mass of the toast will approximately fall by $y = ½g(T/2)^2 = 1.2$ m. Therefore, the only way to save the carpet from the toast "butter face" would be increasing the table height.

It is interesting to note that angular speed is uniform for the first part of the trajectory, when weight is the dominant force. As mg is applied on the center of mass, it does not exert any torque, and the angular momentum of the falling toast remains constant. This approximation fails in the last part of the trajectory, when drag forces become relevant, and the experimental data shown that angular speed increases.