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Abstracts Book

Contents

Global Saddle for symmetric planar maps

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The study of global dynamics has long been of interest. Particular attention has been given to the question of inferring global results from local behaviour, when a unique fixed point is either a local attractor or repellor.

The presence of symmetry in a dynamical system creates special features that may be used to obtain global results. Planar dynamics with symmetry, when the fixed point is either an attractor or a repellor, has been addressed in [1]-[3]. These results ignore the important case when the fixed point is a local saddle. However, in [3] it is shown that the only symmetry groups that admit a local saddle are $Z_2(\langle -Id \rangle)$, $Z_2(\langle \kappa \rangle)$ and $D_2 = Z_2^a \oplus Z_2^b$. The superscripts a and b indicate that the groups Z_2^a and Z_2^b are generated by two reflections, a and b, on orthogonal lines.

We have strong interest in the dynamics of planar diffeomorphisms having a unique fixed point that is a hyperbolic local saddle. We study conditions under which the fixed point is a global saddle. In this talk we address the special case of D_2-symmetric maps, for which we obtain sufficient conditions even if the map is only a C^1 homeomorphism.

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Combinatorial dynamics of strip patterns of quasiperiodic skew products in the cylinder

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We extend the results and techniques from [1] to study the combinatorial dynamics (*forcing*) and entropy of quasiperiodically forced skew-products on the cylinder. For these maps we prove that a cyclic permutation τ forces a cyclic permutation ν as interval patterns if and only if τ forces ν as cylinder patterns. This result gives as a corollary the Sharkovskiĭ Theorem for quasiperiodically forced skew-products on the cylinder proved in [1].

Next, the notion of s-horseshoe is defined for quasiperiodically forced skew-products on the cylinder and it is proved, as in the interval case, that if a quasiperiodically forced skew-product on the cylinder has an s-horseshoe then its topological entropy is larger than or equals to $\log(s)$.

Finally, if a quasiperiodically forced skew-product on the cylinder has a periodic orbit with pattern τ, then $h(F) \geq h(f_\tau)$, where f_τ denotes the *connect-the-dots* interval map over a periodic orbit with pattern τ. This implies that if the period of τ is $2^n q$ with $n \geq 0$ and $q \geq 1$ odd, then $h(F) \geq \frac{\log(\lambda_q)}{2^n}$, where $\lambda_1 = 1$ and, for each $q \geq 3$, λ_q is the largest root of the polynomial $x^q 2x^{q 2} 1$. Moreover, for every $m = 2^n q$ with $n \geq 0$ and $q \geq 1$ odd, there exists a quasiperiodically forced skew-product on the cylinder F_m with a periodic orbit of period m such that $h(F_m) = \frac{\log(\lambda_q)}{2^n}$. This extends the analogous result for interval maps to quasiperiodically forced skew-products on the cylinder.

Moreover, there is a natural question that arises in this setting: *Does Sharkovskiĭ Theorem holds when restricted to curves instead of general strips?*

We answer this question in the negative by constructing a counterexample: We construct a map having a periodic orbit of period 2 of curves (which is, in fact, the upper and lower circles of the cylinder) and without any invariant curve.

In particular this shows that there exist quasiperiodic skew products in the cylinder without invariant curves.

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Statistical stability for piecewise expanding multidimensional transformations

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We shall introduce the concept of statistical stability and offer a guided tour to some results illustrating this concept in certain contexts as quadratic transformations, Viana transformations, Hnon diffeomorphisms or Lorenz flows. We shall consider in more detail a recent result with A. Pumario and E. Vigil in which we give sufficient conditions for the statistical stability of certain classes of piecewise expanding transformations.

Thue-Morse system of difference equations revisited

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The Thue-Morse system of difference equations was introduced in [**?**] as a model to understand the electric behavior (conductor or insulator) of an array of electrical punctual positive charges occupying positions following a one dimensional distribution of points called a *Thue-Morse chain* which it is connected to the sequence **t** = (0110100110010...) called also the *Thue-Morse sequence*. Unfolding the system of difference equations, we obtain the two-dimensional dynamical system in the plane given by

$$F(x, y) = (x(4 - x - y), xy)$$

The interest of such system was stated by A.Sharkovskiǐ as an open problem and proposing some questions.

The most interesting dynamics of the system is developed inside an invariant plane triangle, where hyperbolic periodic points of almost all period appear, there are subsets of transitivity and invariant curves of spiral form around the unique inside fixed point. We have proved that the set of periodic orbits is not dense inside the triangle and does not exist an attractor in Milnor sense.

In this talk we will present also some recent results concerning the behavior of all points outside the triangle, completing the known dynamics of the system. In fact we have obtained that outside the triangle, the orbits of all points are unbounded. Some of them go to infinite in an oscillating way occupying the second and third quadrant of the plane and others are going in a monotone way to infinite. Outside the triangle there are no periodic points. Such new results has an interesting interpretation in terms of the physics of the problem. Additionally we will answer some of the questions stated by Sharkovskiǐ concerning the inside of the mentioned triangle.

We will also present graphical analysis of the evolution of some orbits and also the visualization of the dynamics of the system inside the triangle.

Additionally we will present and comment results on another system of difference equations associate to Fibonacci sequence whose unfolding in $\mathbf{R}^3$ is

$$F(x, y, z) = (y, z, yx - z)$$

On the chaos game of iterated function systems

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Within fractal geometry, iterated function systems provide a method for both generating and characterizing fractal images. An *iterated function system* (IFS) can also be thought of as a finite collection of functions which can be applied successively in any order. Attractors of this kind of systems are self-similar compact sets which draw any iteration of any point in an open neighborhood of itself. There are two methods of generating the attractor: *deterministic*, in which all the transformations are applied simultaneously, and *random*, in which the transformations are applied one at a time in random order following a probability. The *chaos game*, popularized by Barnsley [1], is the simple algorithm implementing the random method. We have two different forms to run the chaos game. One involves taking a starting point and then choose randomly the transformation on each iteration accordingly to the assigned probabilities. The other one starts choosing a random order iteration and then applying this orbital branch anywhere in the basin of attraction. The first form of implementation is called *probabilistic chaos game* [2, 5]. The second implementation is called *deterministic chaos game* (also called *disjunctive chaos game*) [3, 4]. In this paper we show that every IFS of continuous maps on a first-countable Hausdorff topological space satisfies the probabilistic chaos game (see also [5]) and give necessary and sufficient conditions to get the deterministic chaos game. As an application we obtain that an IFS of homeomorphisms of the circle satisfies the deterministic chaos game if and only if it is forward and backward minimal. This provides examples of attractors that do not satisfy the deterministic chaos game. We also prove that every contractible attractor (in particular strong-fibred attractors) satisfies the deterministic chaos game.

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Breakdown of heteroclinic connections and Shilnikov Bifurcations in the Hopf-zero singularity

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If one considers conservative (i.e. one-parameter) unfoldings of the so-called Hopf-zero singularity, one can see that the truncation of the normal form at any finite order possesses two saddle-focus critical points with a one- and a two-dimensional heteroclinic connection. The same happens for non-conservative (i.e. two-parameter) unfoldings when the parameters lie on a certain curve.

However, considering the whole vector field, one expects these heteroclinic connections to be destroyed. This fact can lead to the birth of a homoclinic connection to one of the critical points, producing thus a Shilnikov bifurcation. For the case of $\mathcal{C}^\infty$ unfoldings, this was proved by Broer and Vegter during the 80's, but for analytic unfoldings it has remained an open problem. Recently, under some assumptions on the size of the splitting of the heteroclinic connections, Dumortier, Ibáñez, Kokubu and Simó proved the existence of Shilnikov bifurcations in the analytic case.

Our study concerns the splitting of the one and two-dimensional heteroclinic connections. These cannot be detected in the truncation of the normal form at any order, and hence they are expected to be exponentially small with respect to one of the perturbation parameters. We shall present asymptotic formulas of these splittings, putting emphasis on the differences between the conservative and non-conservative cases. In particular, we prove that under generic conditions, the assumptions made by Dumortier, Ibáñez, Kokubu and Simó hold, so that a Shilnikov bifurcation takes place indeed.

Orbital-reversibility of planar dynamical systems

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We will give necessary conditions for the orbital-reversibility for a class of planar dynamical systems. Based in these conditions, we formulate a suitable algorithm to detect orbital-reversibility which is applied to a family of nilpotent systems and to a family of degenerate systems.

More concretely, we consider a planar autonomous system of differential equations having an equilibrium point at the origin given by

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}), \tag{1}$$

where $\mathbf{x} = (x, y)^{\mathrm{T}} \in \mathrm{R}^2$. We study if it admits some reversibility modulo $\mathcal{C}^\infty$-equivalence.

The problem of determining if system (1) has some reversibility is consider in [1] and [3]. In [2], we are concerned with the *orbital–reversibility* problem: a system is called *orbital–reversible* if there exists some time-reparametrization such that the resulting system admits some reversibility; and our goal is to determine, in the planar case, conditions on the system to be orbital–reversible. As with the reversibility, the presence of some orbital–reversibility is useful in the understanding of the dynamical behavior of the system, because the time-reparametrizations do not change the orbits but only the speed in which they are traversed in time.

For planar systems, there is a strong connection between the center problem and the reversibility property of a planar system: if the system has a non-degenerate center at the origin, then it is reversible, see [5].

The orbital–reversibility property is also closely related to the center problem. For instance, the existence of an orbital reversibility in a monodromic vector field ensures the presence of a center. Also, if a planar system has a nilpotent center at the origin, then it is orbital–reversible, see [4].

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Surfing simplicity

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The *relative equilibria* of n bodies in $\mathbb{R}^3$ submitted to the Newton attraction are certainly the simplest possible solutions of the equations of motion. They exist only for very special configurations, the so-called *central configurations* whose determination is a very hard problem as soon as the number of bodies exceeds 3. The motions are periodic and necessarily take place in a fixed plane.

Things become more interesting if one allows the dimension d of ambient space to be greater than 3: in a higher dimensional space, a relative equilibrium is determined not only by the initial configuration but also by the choice of a hermitian structure on the space where the motion takes place; moreover, if the configuration is *balanced* but not central, the motion is in general quasi-periodic.

Ill address the following questions: what are the possible frequencies of the angular momentum of relative equilibria of a given central (or balanced) configuration and at which values of these frequencies bifurcations from periodic to quasi-periodic relative equilibria do occur?

The entropy spectrum of Lyapunov exponents in non-hyperbolic skew-products

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We consider transitive skew-products dynamics containing (and thus mixing) hyperbolic sets of different indices. Jointly motivated by the examples of robustly transitive diffeomorphisms, dynamics arising in the unfolding of heterodimensional cycles and porcupine-like horseshoes, we identify some abstract properties that enable us to describe the topological entropy of level sets for central Lyapunov exponents.

One of the ingredients is a restricted variational principle to describe the topological entropy of certain level sets taking in consideration measures in a certain subset of hyperbolic measures only.

Existence of inverse integrating factor

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In this work, we study the existence of an inverse integrating factor for a class of systems, in general non-analytically integrable, whose lowest-degree quasi-homogeneous term is a Hamiltonian system and its Hamiltonian function only has simple factors over $\mathbb{C}[x, y]$. That is, we deal with systems of the form

$$\dot{\mathbf{x}} = \mathbf{X}_h + \text{q-h.h.o.t.} \tag{1}$$

For this task, firstly, we calculate a formal orbital equivalent normal form of system (1), i.e. an expression of this system after a change of state variables and a reparameterization of the time, and we focus our study in systems (1) which are formally orbital equivalent systems to

$$\dot{\mathbf{x}} = \mathbf{X}_h + \mu \mathbf{D}_0, \text{ with } \mu = \sum_{j>r} \mu_j,\ \mu_j \in \text{Cor}(\ell_j) \text{ and } h \in \mathcal{P}^{\mathbf{t}}_{r+|\mathbf{t}|} \tag{2}$$

where ℓ_j is the Lie derivative of the lowest degree quasi-homogeneous term of (1).

They are a wide class of these systems, for example, systems with linear part non-null (such as nilpotent systems) and some generalized nilpotent, among others. From Algaba et al. [1], systems formally orbital equivalent to systems (2) are analytically integrable if and only if $\mu \equiv 0$ and, in such a case, they have a first integral of the form $h+$q-h.h.o.t., and consequently, they have an inverse integrating factor. Therefore, for $\mu \not\equiv 0$, systems orbitally equivalent to systems (2) do not have any analytic first integral (non-integrable systems). We study this systems, which are non-integrable, and we show our principal result. In it we give necessary and sufficients conditions for the existence of an inverse integrating factor for systems (2) and we apply this result for the characterization the existence of an inverse integrating factor for some families of generalized nilpotent systems (see [2]). Other works in this way can be seen in Algaba et al. [3, 4].

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A local non-integrability criterion

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We consider the problem of characterizing, for certain natural number m, the local $\mathcal{C}^m$-non-integrability near elliptic fixed points of smooth planar measure preserving maps. Our criterion relates this non-integrability with the existence of some Lie Symmetries associated to the maps, together with the study of the finiteness of its periodic points. Our main result is:

Theorem *Let F be a $\mathcal{C}^{2n+2}$-planar map defined on an open set $\mathcal{U} \subseteq \mathbb{R}^2$ with an elliptic fixed point p, not $(2n+1)$-resonant, and such that its first non-vanishing Birkhoff constant is $B_n = i\, b_n$, for some $0 < n \in \mathbb{N}$ and $b_n \in \mathbb{R} \setminus \{0\}$. Moreover, assume that F is a measure preserving map with a non-vanishing density $\nu \in \mathcal{C}^{2n+3}$. If, for an unbounded sequence of natural numbers $\{N_k\}_k$, F has finitely many N_k-periodic points in $\mathcal{U}$ then it is not $\mathcal{C}^{2n+4}$-locally integrable at p.*

This criterion can be applied to prove that the Cohen map

$$F(x,y) = \left(y, -x + \sqrt{y^2+1}\right),$$

is not $\mathcal{C}^6$-locally integrable at its fixed point. Similarly we obtain non-integrability results for rational maps of the forms

$$F(x,y) = \left(y, \frac{f(y)}{x}\right) \quad \text{and} \quad F(x,y) = (y, -x + f(y)). \tag{1}$$

One of the steps in the proof uses next result about the regularity of the period function on the whole period annulus for non-degenerate centers, question that we believe that is interesting by itself.

Theorem *Let X be a $\mathcal{C}^k$-vector field with $1 \leq k \in \mathcal{N} \cup \{\infty, \omega\}$ with a non-degenerate center p, and let $\mathcal{V}$ be its period annulus. Then the period function T is of class $\mathcal{C}^k$ on $\mathcal{V} \setminus \{p\}$ and, at p, it is of class $\mathcal{C}^{k-1}$, where for the sake of notation $\infty - 1 = \infty$ and $\omega - 1 = \omega$. Moreover, in general, the regularity of T at p can not be improved.*

For controlling the number of periodic points of a given period we use the following proposition:

Proposition *Let $G : \mathbb{C}^N \to \mathbb{C}^N$ be a polynomial map of degree d. Let G_d denote the homogenous map corresponding to the degree d terms of G. If $\mathbf{y} = \mathbf{0}$ is the unique solution in $\mathbb{C}^N$ of the homogeneous system $G_d(\mathbf{y}) = \mathbf{0}$, then the polynomial system $G(\mathbf{y}) = \mathbf{0}$ has finitely many solutions.*

Connectivity of Julia sets of Newton maps: A unified approach

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We give a unified proof of the fact that the Julia set of Newton's method applied to a holomorphic function of the complex plane (a polynomial of degree large than 1 or an entire transcendental function) is connected. The result was recently completed by the authors' previous work [1], as a consequence of a more general theorem whose proof spreads among many papers, which consider separately a number of particular cases for rational [2] and transcendental maps, and use a variety of techniques. In this note we present a unified, direct and reasonably self-contained proof which works for all situations alike.

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Irregular behaviour of invariant curves

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In this talk we will focus on invariant curves of quasi-periodically forced maps,

$$\left.\begin{array}{rcl}\tilde{x} & = & f(x,\theta,\mu),\\ \tilde{\theta} & = & \theta+\omega,\end{array}\right\} \tag{1}$$

where $x \in \mathbb{R}^n$, $\theta \in \mathbb{T}$, ω is Diophantine and μ is a real parameter. The map f is assumed to be of class C^r, $r \geq 1$. An invariant curve is a C^1 map $\theta \mapsto x(\theta)$ such that $f(x(\theta),\theta,\mu) = x(\theta+\omega)$. Assume that, for a given value of the parameter $\mu = \mu_0$, (1) has an attracting invariant curve, and that when μ goes from μ_0 to a critical value μ_1 this Lyapunov exponent goes to zero. We are interested in the possible behaviours of the invariant curve when μ approaches μ_1. In particular, we are interested in fractalization phenomena that might give rise to the appearance of a Strange Non-Chaotic Attractor. To study this phenomenon we will focus on a simpler situation, given by the affine system

$$\left.\begin{array}{rcl}\tilde{x} & = & \mu A(\theta)x+b(\theta),\\ \tilde{\theta} & = & \theta+\omega,\end{array}\right\} \tag{2}$$

where $x \in \mathbb{R}^2$. Moreover, we will assume that the corresponding linear system

$$\left.\begin{array}{rcl}\tilde{x} & = & \mu A(\theta)x,\\ \tilde{\theta} & = & \theta+\omega,\end{array}\right\}$$

is non-reducible. A remarkable example of a non-reducible system is given by

$$A(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

We will show that (2) has an invariant curve that displays a fractalization process when μ goes to a critical value.

On the equilibrium points of an analytic differentiable system in the plane. The center–focus problem and the divergence.

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We shall recall briefly how can be the local phase portraits of the equilibrium points of an analytic differential system in the plane, and we shall put our attention in the center-focus problem, i.e. how to distinguish a center from a focus. This is a difficult problem which is not completely solved. We shall provide some new results using the divergence of the differential system.

documentclassMeetingAbstracts

Weight vectors of planar quasi-homogeneous differential systems

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The planar differential system $\dot{x} = P(x,y),\ \ \dot{y} = Q(x,y)$, with $P, Q \in \mathbf{C}[x,y]$, is called *quasi–homogeneous* if there exist s_1, s_2, $d \in \mathbf{N}$ such that for an arbitrary $\alpha \in \mathbf{R}^+$, it is verified that $P(\alpha^{s_1}x, \alpha^{s_2}y) = \alpha^{s_1-1+d}P(x,y)$ and $Q(\alpha^{s_1}x, \alpha^{s_2}y) = \alpha^{s_2-1+d}Q(x,y)$. The quasi-homogeneous systems have important properties (for example, all of them are integrable) and they have been studied from many different points of view (integrability, centers, normal forms, limit cycles). Recently, an algorithm for constructing all the nonhomogeneous quasi-homogeneous polynomial differential systems of a given degree was published in [1]. Using this algorithm, the article authors obtained the classification of the nonhomogeneous quasi-homogeneous planar systems of degree 2 and 3, and later other authors solve the case for degree 4 (see [4]) and 5 (see [5]). Now, in our work [2], we give the classification of quasi-homogeneous systems on the basis of the weight vector concept, especially in terms of the minimum weight vector w_m which it is proved to be unique for any quasi-homogeneous system. Later, we obtain the exact number of different forms of quasi-homogeneus but nonhomogeneous planar differential systems of an arbitrary degree n, proving a nice relation between this number and the Euler's totient function, whose definition and properties can be seen in [3]. Finally, we provide software implementations for some of the results discussed above.

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A homotopical property of attractors

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Given a homeomorphism h of $\mathbb{R}^3$, we say that a compact subset K of $\mathbb{R}^3$ is an attractor if it is invariant, Lyapunov stable and there is a neighborhood $U = U(K) \subset \mathbb{R}^3$ such that all orbits starting at U converge to the set K. This definition leads to the following question: what compact sets can be realized as (local) attractors of some homeomorphism?

We will construct a topological torus $\mathcal{T} \subset \mathbb{R}^3$ that cannot be an attractor. To this end we show that, given an attractor K, the fundamental group of $\mathbb{R}^3 \setminus K$ has certain finite generation property that the complement of $\mathcal{T}$ does not have.

Center boundaries for planar piecewise differential systems of two zones

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The periodic orbits that appear in a piecewise differential system are worthy of an special attention, as it happens in the classical case. In particular, there are many published references studying the number and location of limit cycles. See, for example, [1] in planar systems and [2] about systems in higher dimensional space.

In the study of piecewise differential systems with two zones, [3] shows the importance of the boundary between the two zones when you analyze the limit cycles. From this idea, we characterize the conditions of the boundary between two given differential systems in order to obtain different behaviors of the periodic orbits. For example, given a pair of linear planar systems of certain type, we can construct a 1-dimensional differentiable manifold such that any orbit is a periodic orbit. It means, given a pair of linear systems, we can consider a nice boundary between them such that we have a global center. So, if we know this central boundary, we can determine the limit cycles of a piecewise differential system with a fixed boundary as the intersections of this curve and the central boundary.

The existence of this boundary does not imply that it would be explicitly known except for some particular families of planar systems. Anyway, we are interested in more general properties of these curves and not just their explicit expressions. One of our main propose is to find a way to characterize these curves in a quite general situation.

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A strategy to locate fixed points and global perturbations of ODES: Mixing Topology with Metric Conditions

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It is well known that if $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is an orientation preserving embedding with a recurrent fixed point then f has a fixed point. However, we have no information on its location. In this setting we propose the following strategy: if, for simplicity, p is a periodic point of order n and γ is any arc joining p with $f(p)$, then f has a fixed point in γ or in a bounded connected component of

$$\mathbb{R}^2 \backslash \gamma \cup \cdots \cup f^{n-1}(\gamma).$$

For $n = 2$, the proof of the previous strategy was given by Morton Brown in [1] and partial results for $Id + K$ with K a contraction were given by Bonatti and Kolev in [3]. As mentioned in Campos and Ortega [2], these type of strategies allows us to infer many dynamical properties in planar dynamical systems. As a particular application, consider

$$x'' + x = f(t, x, x') \tag{1}$$

with f bounded and 2π-periodic in time. If the set of 2π-periodic solutions of (1) is bounded, then the set of $2k\pi$-periodic solutions of (1) is bounded as well for all $k \in \mathbb{N}$.
This is a joint work with G. Graff and the results are contained mainly in [4]

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Return maps, separatrix-like maps, standard-like maps, boundaries of confined motion, the Sitnikov problem

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After reviewing some general settings for return maps in problems reducible to 2D symplectic maps, details on the construction of these maps are presented. Different forms of such maps close to splitted separatrices (separatrix maps) are introduced, taking into account the size and shape of the splitting function and also the return time to the domains of interest, which differs if the fixed points are of hyperbolic or parabolic type. Then it is shown how to derive approximations by suitable standard-like maps. Dynamical consequences concerning the existence of invariant rotational curves (IRC) are derived. An application is made to theoretically estimate the location of the outermost IRC in the Sitnikov problem, which is in good agreement with numerical data. Details are given on the properties of standard-like maps with two harmonic terms, compared to the classical standard map. A method to estimate the amount of chaos depending on the form of the separatrix map is introduced. The systems we consider are assumed to be analytic, despite several of the properties we study are no longer analytic. Some open problems close the presentation.

Superstable periodic orbits, reducibility and renormalization in quasiperiodic perturbations of 1d maps

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Let g_α be a one-parameter family of one-dimensional maps with a cascade of period doubling bifurcations. In this work we deal with the effect of a quasi-periodic perturbation on this cascade. If ϵ is the perturbing parameter, we can prove under generic conditions that from each superattracting point of the unperturbed map, two reducibility loss bifurcation curves (for which the invariant curve changes from reducible to non-reducible) in the parameter plane (α, ϵ) are born. This means that these curves are present for all the cascade. The definition of a suitable renormalization operator allow us to explain the asymptotic behaviour of the slopes of the cited curves when the corresponding period tends to infinity. This is a joint work with À. Jorba and P. Rabassa.

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Repellers of random walks

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In his 2009 thesis at IMPA, Carlos Bocker proved that the Lyapunov exponents of random (2D) products of 2-by-2 matrices always depend continuously on the matrices' coefficients and their probability weights. The proof is based on a detailed analysis of the dynamics of the associated random walk in projective space.

Most recently, Avila, Eskin and the speaker announced that they are able to carry this analysis to arbitrary dimension, using a very different (cost functions) approach. Thus, continuity of Lyapunov exponents on the underlying data holds in full generality for iid random products of matrices.

This new approach has been extended in the thesis of Elaís Malheiro to prove that the 2-dimensional statement generalizes to Markov products of matrices. Moreover, it is in the basis of the work of Lucas Backes, another 2014 thesis at IMPA, which contains substantial progress towards proving that continuity of Lyapunov exponents holds for very general 2-dimensional Holder cocycles over hyperbolic systems.

These results are in stark contrast with observations of Ricardo Mañé in the 1980's, completed by Jairo Bochi and the speaker two decades later, according to which one can often annihilate the Lyapunov exponents of continuous linear cocycles, thus making continuity a very particular situation in that context.

Expanding Baker Maps: A First Tool To Study Homoclinic Bifurcations of 3-D Diffeomorphisms

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It is known that if a one-parameter family of two-dimensional diffeomorphisms unfolds a homoclinic tangency, then a family of limit return maps can be constructed. This family is closely related to the one-dimensional quadratic map $f_a(x) = 1 - ax^2$.

In [2], the author defines the family of limit return maps $T_{a,b}(x, y) = (a+y^2, x+by)$ associated to families of three-dimensional diffeomorphisms unfolding homoclinic tangencies. In [3], the authors make an exhaustive numerical analysis for the family $T_{a,b}$ that shows the (possible) existence of strange attractors with one and two positive Lyapunov exponents. In order to demonstrate that these strange attractors really exist we have constructed certain two-dimensional piecewise linear maps, called *Expanding Baker Maps (EBMs)*, exhibiting the same kind of attractors that of $T_{a,b}$. The aim of this talk will be to explain how these *EBMs* arise and their link with the family $T_{a,b}$, in addition we will demonstrate that *EBMs* display strange attractors and a unique ergodic absolutely continuous invariant measure (this part is included in [4] and [5]). Finally, we will prove that our family of maps is *statistically stable* (see [1]). [1]

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